# Deformable Objects Manipulation Using Model Adaptation Techniques

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Abstract—Precise manipulation of deformable objects remains challenging due to the trade-off between the model complexity and accuracy. Pushing has been a foundational task in demonstrating nonprehensile manipulation capability with robots. In this work, we study the problem of pushing a deformable object. To bridge the gap between complexity and accuracy, we propose an adaptation to the rigid body model for representing the motions of a deformable object. We concentrate on the trajectory tracking problem in which a robot arm pushes a deformable object along a desired trajectory. To investigate the utility of the proposed model, we introduce two control methods. First, an adaptive model predictive control (MPC) is considered to handle the uncertainties caused by the deformations of the object. Second, a learning-based control strategy aimed at achieving improved performance by predicting the object's deformation has been developed. Our results demonstrate the efficiency of the proposed techniques by decreasing the deviation from the desired trajectory by more than 50%.

## I. INTRODUCTION

As robot deployments continue to increase in novel applications from manufacturing to healthcare, manipulation of deformable objects remains to be a challenging research problem. To tackle this problem, various model-based and learning-based techniques are utilized [1]–[5]. Model-based approaches provide a wide range of accuracy [6]. However, extending available models for different deformable objects is hard to attain due to their complex, anisotropic, and nonlinear mechanical behavior. Learning-based approaches are popular owing to their capability of handling multi-dimensional problems. Nonetheless, these methods require quality data to provide satisfactory and reliable results.

Pushing is well-studied in the literature and it serves as a foundational capability for more complex manipulation tasks [7]. Applications of pushing include insertion of a cable to the socket [8], pre-grasp manipulation [9], and creating a corridor to the robot's target in jammed environments [7].

Manipulation of deformable objects has been the focus of recent research. [10] describes different classes of deformable objects and their applications in robotics. Fig. 1 demonstrates an example of deformations in a soft object. [11] proposed a scheme for learning force control from demonstration for deformable object manipulation. However, this method is not applicable to applications that require real-time trajectory planning due to its computational cost. [12] compares the performance of reinforcement learning and learning from



Fig. 1: Soft object deformation under different applied forces.

demonstration in soft tissue manipulation for surgical applications. Nonetheless, this approach is limited to 2D manipulation of deformable objects and vulnerable to the visual occlusion of the work space.

[6] conducts a survey on the model-based manipulation planning methods as applied to deformable objects. [5], [8], [13] propose a 3D geometric model for linear flexible objects, which enables task automation in manipulating these objects both in simulation and real world. Nonetheless, these methods do not satisfy the accuracy requirement in many applications. On the other hand, models from the finite element method (FEM) are demonstrated to perform accurately in applications [14], [15]. Still, the high computational cost in these methods makes them applicable only for off-line simulation and planning purposes. Predicting deformations of a soft object is a hard problem. To tackle this problem, an adaptive approach is suggested in [16] to automatically control deformations in a model-free manner which is restricted to objects with a well-defined texture. MPCs are also beneficial in manipulating deformable objects since they make no prior assumptions on linearity [17]-[19].

In this work, we study the problem of pushing a deformable planar object along a desired trajectory on a flat surface by a robotic manipulator. To this end, we calculate optimal actions for the robot to minimize the deviation of the object's center of mass from the target trajectory. The main contributions of this paper are: (1) proposing a new deformable object motion model obtained by adaptation from existing rigid object models; (2) designing an adaptive MPC to enable the robot to interact with the deformable object on a flat surface by estimating the varying parameters in the object's motion model; (3) introducing a learning-based control strategy to approximate the object's deformation using a multi-layer perceptron [20] regression unit. Comparative results are provided in the experiment environment to validate the quality of results.

## II. SLIDER MOTION MODEL

In this section, the motion model for both rigid and deformable slider objects is presented. Two contact interactions, namely sticking and sliding, are possible at the contact point between the pusher and slider object. Accordingly, the pusherslider system is considered a hybrid dynamical system [19], [21]. In this work, we focus on sticking mode between the robot and the object. The equations of motion are obtained under the quasi-static assumption [19], [22].

## A. Slider Kinematics

The rigid slider kinematics is represented in Fig. 2 while the velocity  $\mathbf{u} = \begin{bmatrix} u_n & u_t \end{bmatrix}^{\mathbf{T}}$  is applied to it via a single contact point [19], [21]. The pose of the object in 2D is defined by  $\mathbf{x} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{\mathbf{T}}$  where x and y denote the position of the center of mass and  $\theta$  is the rotation of the object relative to the inertial reference frame  $(F_a)$ . The contact point position relative to the object's center of mass (C.O.M) is described by  $\mathbf{p} = \begin{bmatrix} p_x & p_y \end{bmatrix}^{\mathbf{T}}$  in object's reference frame  $(F_b)$ .



Fig. 2: (a) Kinematics of the rigid slider object in a single point of contact with robot pusher. Dashed green arrow indicates the region for sticking to the contact point, and solid red arrows show sliding zones. (b) Changes in the C.O.M and outer shape of a deformable slider

## B. Rigid Slider Equations of Motion

To map the applied frictional forces on the object to its resulting velocity, we use ellipsoidal approximation under the quasi-static assumption. The unconstrained motion equations of the pusher and rigid slider system in sticking mode can be expressed by [19], [23]:

 $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ 

(1)

with

$$f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \mathbf{R}^{\mathbf{T}} \mathbf{Q} \\ \mathbf{b} \end{bmatrix} \mathbf{u}, \quad \mathbf{R} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$
$$\mathbf{Q} = \frac{1}{c^2 + p_x^2 + p_y^2} \begin{bmatrix} c^2 + p_x^2 & p_x p_y \\ p_x p_y & c^2 + p_y^2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \frac{-p_y}{c^2 + p_x^2 + p_y^2} & p_x \end{bmatrix}$$

# C. Deformable Slider Equations of Motion

The applied force on a deformable object may significantly change the object's outer shape and center of mass position, as depicted in Fig. 2. Several factors affect the deformation process. These include the force applied to the object, current geometrical traits, and object composition. Moreover, the pusher slider system is associated with uncertainties arising from non-uniform mass distribution of the deformable slider, changeable friction forces between surfaces, and noisy sensor data.

Based on motion equation described by (1), a modified state-space equation is presented in (2) for deformable objects considering both deformation and uncertainties [24].

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) + g(\mathbf{x}, \mathbf{u})\boldsymbol{\sigma}$$
(2)

where **x**, **u** represent system states and control commands, respectively. The term  $g(\mathbf{x}, \mathbf{u})$  represents the deformation model as a function of inputs and states and  $\sigma$  is the system uncertainty.

#### **III. PROBLEM FORMULATION**

Let  $\mathbf{x} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$  and  $\mathbf{u} = \begin{bmatrix} u_n & u_t \end{bmatrix}^T$  be compact sets representing states and control inputs with x, y, and  $\theta$  representing the position and orientation of the object measured at its C.O.M. and  $\mathbf{x}_d$  denoting the desired states. The problem of trajectory tracking can then be formulated by,

min 
$$\sum_{i=0}^{N-1} (\bar{\mathbf{x}}_{i+1}^T M \bar{\mathbf{x}}_{i+1} + \mathbf{u}_i^T L \mathbf{u}_i)$$
(3)

subject to 
$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$
  
 $u_n \ge 0$   
 $u_t \ge \gamma_b u_n$   
 $u_t \le \gamma_t u_n$ 

where  $\bar{\mathbf{x}} = ||\mathbf{x}_d - \mathbf{x}||$  is the object's deviation from the desired trajectory and N is the prediction horizon. The terms M and L are weight matrices associated with the error states and weight matrices associated with control inputs, respectively. Parameters  $\gamma_b$  and  $\gamma_t$  represent motion cone boundaries [19].

#### IV. METHODOLOGY

In this section, we introduce a two-level scenario to undertake the manipulation task for deformable objects. First, an adaptive model predictive approach is suggested to estimate the uncertainties and unpredicted deformations. Second, to decrease vulnerability in adaptive parameters and straighten the estimations, an additional learning-based technique is equipped.

# A. Adaptive Model-Based Predictive Control

Real-time measurements and estimation of geometrical parameters for a deformable object are complex and almost intractable due to consecutive changes in its shape. With this consideration, adopting motion equations for deformable objects from rigid objects with similar shapes seems to be a good alternative. The primary purpose of using an adaptive MPC is to perform a fast and precise estimation of the system's uncertainties. To provide fast adaptation to changes in the deformable object during manipulation, we used the uncertainty estimator introduced by Equation (4). In (4), uncertainty is an unknown random vector that is assumed to be bounded and lies within an initially known compact set. This assumption is rational since applying finite force in a limited time frame is unlikely to change the objects shape drastically [24].

$$\dot{\hat{\mathbf{x}}} = f(\mathbf{x}, \mathbf{u}) + g(\mathbf{x}, \mathbf{u})\hat{\boldsymbol{\sigma}} + k\mathbf{e} + \mathbf{w}\dot{\hat{\boldsymbol{\sigma}}}$$
(4)

# B. Learning-Based Control

In this part, we introduce a learning-based control strategy for the manipulation of deformable objects. The primary goal is to overcome the parameter tuning difficulties in adaptive MPC. The advised method provides not only fast model adaptation but also predicts unforeseeable behaviors in the deformation process. In this case, we calculate the object's expected position and orientation from Equation (1). Moreover, we measure the object's actual rotation and position as well as  $p_x$  and  $p_y$  to calculate the actual deviation from the model for each applied action.



Fig. 3: Model adaptation scheme

# V. RESULTS

We present a performance evaluation for comparing the performance of adaptive MPC and learning-based control in the trajectory tracking problem. Each scenario is replicated twenty times to test repeatability and determine a deviation bound from the desired trajectory. We utilize a plastic bag filled with rice as a benchmark deformable object in our experiments.

## A. Pushing a deformable slider using adaptive MPC

This section demonstrates adaptive MPC's performance in the tracking problem, where the pusher guides the deformable slider object to follow the desired trajectory. As illustrated in Fig. 4, although adaptive MPC performs satisfactorily initially, it cannot provide proper control commands when the deviation is greater than a specific threshold. This insufficient performance mainly arises from inaccuracy in  $g(\mathbf{x}, \mathbf{u})$  modelling. Moreover, a major drawback of this method is the difficulty in tuning controller parameters.

#### B. Pushing a deformable slider using learning-based MPC

In this part, performance of learning-based MPC is investigated. As depicted in Fig. 4, learning-based MPC results in a narrower deviation bound compared to adaptive MPC. This improvement in results is due to learned deformations instead of approximate functions.



Fig. 4: Pushing a deformable object using (a) adaptive MPC and (b) learning-based MPC controller. Dashed red line, dash-dotted blue line, and solid green line indicate desired path, average actual path and best actual path respectively.

### VI. CONCLUSION

In this work we developed a model adaptation to represent the motions of a deformable object to demostrate robust pushing tasks. In addition, we suggest an adaptive MPC and a learning-based control strategy for the trajectory tracking problem. The validity of methods is investigated for the trajectory tracking problem in two different scenarios.

The proposed control schemes are a prototype with some restrictions that suggest directions for future works. Applying online learning approaches can increase the control system's flexibility facing different deformable slider objects. Furthermore, providing force feedback and more accurate perception methods to deliver a precise 3D estimation of an object's shape can result in more accurate outcomes.

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